A Hierarchy of South Korean Elementary Teachers Knowledge for Teaching Mathematics

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Abstract:
The purpose of this study is to develop a framework for understanding teachers’ knowledge for teaching mathematics (MKT) by examining a hierarchy of 317 South Korean elementary teachers’ knowledge for teaching mathematics. Through a web-based survey, the teachers were asked to rank order their beliefs about and use of knowledge subdomains for teaching mathematics. Findings show that teachers’ beliefs about knowledge for teaching mathematics are divergent from current research trends. Also, their beliefs about the subdomains of knowledge for teaching mathematics were ranked differently in comparison to their actual use of the subdomains. Their hierarchy for MKT presents suggestions for the type of curriculum that might be offered to teachers to develop their knowledge at the pre- and in-service level.

Keywords:
Knowledge for Teaching Mathematics; Mathematical Knowledge for Teaching; Beliefs about Knowledge for Teaching Mathematics; Pedagogical Content Knowledge

1. INTRODUCTION

Since the No Child Left Behind Act took effect in 2001, diverse approaches have been applied to understand and define teachers’ knowledge for teaching mathematics, which affect students’ learning [1–4]. This body of research suggests that to increase the quality of teachers’ knowledge for teaching mathematics, improvement is needed in the preparation of new and practicing mathematics teachers. Most importantly, policymakers and educators are concerned with how teachers might promote students’ high academic achievement [5]. This concern is not unique to the U. S. education system. Park argues that globally, teachers are viewed as the “driving force behind the improvement in educational quality and the effort to improve student mathematical achievement” ([6], p. 181).

Teachers’ knowledge for teaching mathematics does not simply mean that teachers have knowledge of mathematical content, such as algebra and geometry [7]. Teaching is not just delivering procedural information; rather it is also about helping students develop conceptual understanding [8]. Albert asserts that “teachers need to develop not just a deeper knowledge of subject matter but an understanding of the mathematical process of inquiry to enrich their teaching practices and to encourage critical thinking skill development in their students” ([9], p. 31). In addition, the Common Core State Standard for Mathematics
also proposes that the teaching of mathematical content needs to emphasize both procedural skills and conceptual understanding to ensure that students are learning and engaging in the content and information they need to succeed at higher levels [10].

However, although the studies presented above name and define teachers’ knowledge in various ways (e.g., knowledge of content or teachers’ knowledge of students’ mathematical thinking), it is not easy to understand teachers’ use of their knowledge for teaching mathematics in their teaching practices [11]. The major issue of studies about teachers’ knowledge for teaching mathematics is that most of these studies only focus on pedagogical content knowledge (PCK). Shulman defines PCK as one of the domains of teachers’ knowledge for teaching mathematics. PCK indicates a pedagogical transformation of mathematics concepts that consider how mathematics content will be taught and how teachers might understand student learning [12, 13]. Yet, there is a lack of empirical evidence that teachers use PCK over other categories of knowledge for teaching mathematics or that teachers’ knowledge of PCK warrant effective mathematics instruction. In addition, there are limitations regarding the effects of elementary teachers’ PCK for teaching mathematics. While research suggests that elementary teachers’ PCK for teaching mathematics affects most students’ mathematics achievement, there is little evidence of its affect on minority students and students who receive low scores on previous achievement tests (e.g., [7, 14]).

Using information from previous research conducted in the United States regarding mathematics knowledge for teaching (MKT), the purpose of this study is to develop a framework for understanding teachers’ MKT by examining a hierarchy of South Korean elementary teachers’ MKT. We use the six-subdomains of MKT developed from the conceptual framework of this study to explore elementary teachers’ importance of their use of and beliefs about mathematics knowledge for teaching. Our framework calls for teachers to improve their knowledge for teaching mathematics based on the notion that teachers’ knowledge influences classroom performance and student learning. In addition, we assumed that teachers’ mathematics teaching beliefs are adaptive dynamic contracts [15] because their beliefs mediate relationships between knowledge and behaviors interacting within appropriate contexts [16].

This study focuses on elementary teachers of South Korea in order to understand how MKT influences their beliefs and practices. Cai suggests that cross-national research might provide opportunities to understand wide-ranging issues about teaching and learning [17]. Investigating elementary teachers of South Korea may offer implications for researchers, policymakers, and teachers in both countries. With the quality of elementary teachers being highly controlled by the South Korean Government, this study is expected to acquire consistent and reliable results. Furthermore, South Korea’s national curriculum was developed based on the U.S. curriculum, resulting in all elementary teachers in South Korea teaching mathematics according to the National Mathematics Curriculum. This study addresses the following questions based on the results of a survey of 317 South Korean elementary teachers: (1) What subdomains of knowledge do the South Korean elementary teachers use when they teach mathematics? (2) What subdomains of knowledge do the South Korean elementary teachers believe are important for effective mathematics instruction?

To address the research questions posed for this investigation, we begin with a discussion of relevant conceptual studies in the area of elementary teachers’ knowledge for teaching mathematics. These studies provide the foundation for developing the framework of this study. Next, we present an emerging framework that includes our analysis of the conceptual studies to illustrate how this current study extracted the subdomains of knowledge for teaching mathematics. The next section describes the methods and procedures employed for data collection, which includes the educational context of South Korea as well as the quantitative and qualitative techniques used in the analysis of survey items and interview questions. Then, we present the findings based on the results of the analysis of the data. The final section focuses on a discussion of these findings and the importance of them to research and practice.
Obtaining common subdomains of MKT is the key in conducting surveys to examine the hierarchy of South Korean elementary teachers’ MKT. This section investigates the major conceptual studies about elementary teachers’ knowledge for teaching mathematics. The discussion in this section will provide an overall understanding of the subdomains of knowledge for teaching mathematics. Since most South Korean research on teachers’ MKT has been conducted based on the U.S. research framework [18–20], we concentrate on the conceptual studies of the categories or domains of MKT in the United States. The domains from four significant studies are examined for this current study: major categories of teachers’ knowledge [21], components of mathematics teachers’ knowledge [22], central domains of teachers’ knowledge for teaching mathematics [23], and technological pedagogical content knowledge [24].

In 1986, Shulman provided a framework for teachers’ knowledge for teaching that has remained mostly unchanged, despite the increasing number of studies on teachers’ knowledge for teaching (e.g., [25–27]). This framework has had a huge influence on understanding the categories of teachers’ knowledge for teaching mathematics, although Shulman did not specify any categories about this knowledge (e.g., [23, 28, 29]). Shulman’s key categories of teachers’ knowledge for teaching are presented in Figure 1.

Effective teachers acquire in-depth knowledge of how to represent the subject matter to students [30]. Shulman named this profound knowledge pedagogical content knowledge (PCK) [21]. PCK is a teachers’ special form of professional understanding, which provides a special blend of content and pedagogy [12]. Although all are included, PCK is not limited to useful representations, unifying ideas, clarifying examples and counter examples, helpful analogies, important relationships, and connections among ideas [31]. PCK should also contain the knowledge and characteristics of learners, knowledge of educational contexts, knowledge of educational ends, purposes, and values, including their philosophical and historical bases [12], and knowledge of how to transform content into forms that are adaptive to the variations in ability and background presented by the students [13].

Based on Shulman’s categories of teachers’ knowledge, Fennema and Franke specified components of mathematics teachers’ knowledge as presented in Figure 2 [22].

Knowledge of mathematics indicates conceptual understanding of mathematics including content knowledge. Fennema and Franke argue that there is a positive correlation between a teacher’s conceptual understanding of mathematics and the quality of classroom instruction; therefore, it is important that teachers have knowledge of mathematics [22]. Fennema and Franke relabeled PCK calling it knowledge of mathematical representations [22]. Since mathematics is seen as a composition of a large set of highly related abstractions, teachers’ knowledge of mathematical representation is significant to students developing a clear understanding of mathematical concepts [32]. Fennema and Franke stated, “If teachers
do not know how to translate those abstractions into a form that enables learners to relate the mathematics to what they already know, they will not learn with understanding” ([22], p. 153). Knowledge of students includes teachers’ understanding of their students and the educational context in which students are located [22]. According to Fennema and Franke [22], learning is based on what happens in the classroom. Therefore, it is important for teachers to have knowledge of the learning environment and of what students do. Fennema and Franke argued that teachers’ beliefs, knowledge, judgments, and thoughts might affect the decisions regarding teachers’ plans and actions in the classroom [22]. They call this category *knowledge of teaching and decision-making*. The process of decision-making in the mathematics classroom may differ based on teachers’ teaching experiences. Thus, intensive investigation is needed in order to define the successful ways for making decisions concerning effective mathematics instruction [33].

In 2008, Ball, Thames and Phelps presented a specified map of teachers’ knowledge for teaching mathematics by rearranging Shulman’s initial categories [23]. These categories are presented in Figure 3.

![Figure 3. Key Domains of Teachers Knowledge for Teaching Mathematics](image)

Ball, et al. distinguished PCK and SMK first and then relocated the other domains of teachers’ knowledge [23]. For PCK, Ball et al. take Shulman’s definition and place three subdomains under PCK: knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum.

In 2006, Mishra and Koehler described a new category of teachers’ knowledge for teaching: *technological pedagogical content knowledge* (TPCK) [24], now known as TPACK. TPACK indicates the integration of SMK, PCK, and the knowledge of technology for teaching and learning [34]. However, TPACK is not just a straightforward combination of SMK, PCK, and technology, but rather, it is transformative knowledge combining each constitution into new forms to maximize the effectiveness of educational technology in the classroom [35, 36]. Teachers also need TPACK when they teach mathematics because
it might affect the quality of instruction [36]. In recent years, TPACK has been accepted in educational research fields, but there is not much evidence of how teachers use TPACK effectively in the classroom [37, 38].

Since Shulman [21] developed categories of teachers’ knowledge for teaching, studies have attempted to reveal what knowledge is needed for teaching mathematics and how this knowledge is related. The importance of these subdomains in this discussion is that teachers’ knowledge consists of basic knowledge and applied knowledge. Teachers should know basic concepts of mathematics and mathematics education (e.g., content knowledge, curriculum knowledge, knowledge of learners) and how to apply their knowledge in their instruction (e.g., PCK or specialized content knowledge). In addition, the studies about teachers’ knowledge for teaching mathematics share the basic assumption that applied knowledge may emerge by comprising categories of basic knowledge. The investigation of teachers’ MKT discussed in this section provides a basis for developing a framework in this study, which will be discussed in the following section.

3. AN EMERGING FRAMEWORK

As noted previously, the conceptual framework for this study is built on the assumption that teachers’ knowledge for teaching mathematics influences classroom performance and student learning [39], and that teachers’ beliefs about mathematics instruction affect their knowledge [7]. Also, this study assumes that teachers use diverse types of knowledge in their mathematics instruction rather than relying on a single subdomain of knowledge during mathematics instruction. Subsequently, defining the subdomains of teachers’ knowledge for teaching mathematics is key to understanding the kinds of knowledge that elementary teachers use and need for their mathematics instruction. In defining teachers’ knowledge for teaching mathematics, this study draws on the work of four conceptual studies that are cited most often by researchers in the literature (i.e. [12, 22–24]). Although these studies attempt to specify teachers’ knowledge domain for teaching mathematics and the definitions of each subdomain, their location and relationships are still controversial. There is a lack of a clear explanation of each subdomain [40]. The distinction between content knowledge and pedagogical content knowledge is vague [29, 41], and the relationship between and among subdomains is not conclusive [42]. Therefore, this study combines subdomains of teachers’ knowledge that relate to each other, although the original studies classify them into different domains.

From the analysis of the conceptual studies, this study extracts common subdomains of teachers’ knowledge for teaching mathematics regardless of the relationship among subdomains, which were defined by the original studies as shown in Figure 4.

As illustrated in Figure 4, this study defines six subdomains of teachers’ knowledge for teaching mathematics: Knowledge of mathematics students, Knowledge of mathematics content, Knowledge of mathematics curriculum, Knowledge of mathematical representations, Knowledge of mathematics educational theory, and Knowledge of mathematical technology. These subdomains were used to broaden our understanding of teachers’ knowledge for teaching mathematics and to develop items for the survey.

4. METHODOLOGY

This study applies mixed-methods, utilizing both survey and interview data collected over a one-year period from May 2012 to May 2013. The National Research Council recommends that investigations may be bolstered considerably by using several approaches that incorporate “quantitative estimates of population characteristics and qualitative studies of localized context” ([43], p. 108). According to
Johnson and Onwuegbuzie, “Mixed methods research offers great promise for practicing researchers who would like to see methodologies described and develop techniques that are closer to what researchers actually use in practice” ([44], p. 15). Therefore, mixed methods research may answer a broader and complete range of research questions than a single method [44, 45]. The first approach was used sequentially to inform the second approach, from which contradictions and new perspectives emerged [45–47].

4.1 Educational Context in South Korea

The quality and distribution of elementary teachers are highly controlled by the government in South Korea. For example, there are only thirteen universities that offer preservice education programs for those interested in becoming elementary school teachers. After graduation, preservice teachers acquire
the teacher certification at Level 2. At this certification level, teachers are qualified to work in private elementary schools. In order to work in a public school, preservice teachers are required to pass the National Teacher Recruitment Examination provided by the province to which they apply. After passing the exam, these teachers are placed at an elementary school by the Office of Education of each province. There are 5,895 elementary schools in South Korea, only 76 (1.3%) of which are private schools [48].

In South Korea, elementary teachers in both public and private schools are required to teach mathematics according to the National Mathematics Curriculum of South Korea. The National Mathematics Curriculum provides mathematical topics that students should learn in each grade, the instructional goal of every topic and how to assess students’ work. Based on the National Common Basic Curriculum of South Korea, the Government develops the mathematics textbooks and teachers’ guidebooks and then provides the materials to all elementary students and teachers in South Korea for free. These guidebooks contain short lesson plans and materials for teaching.

### 4.2 Participants

The target population of the study was South Korean elementary teachers. This study focused on elementary teachers of South Korea because elementary teachers’ quality is highly controlled by the government, and every elementary teacher teaches mathematics based on the National Mathematics Curriculum [49]. This study chose participants who work in Seoul due to its geographical accessibility. Regarding the elementary school in which each participant is working, the location of the school may not be significant, since teachers’ quality and distribution are highly controlled by South Korea’s Ministry of Education. By law, elementary teachers are required to change teaching grades each year within their school and change schools every five years within a province.

According to the Ministry of Education of South Korea [48], there are 181,435 elementary teachers in South Korea, and among them, 29,762 elementary teachers work in Seoul. Surveys were sent to 1,109 elementary teachers, to which 317 elementary teachers responded, giving a response rate of 29%. According to a study by Hamilton (2009), response rates for various types of surveys, including online surveys, average about 26% [50]. Descriptive statistics about the obtained sample are provided in Table 1.
Table 2. Demographic Information of the Interview Participants

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male (n)</th>
<th>Female (n)</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5 years</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>6-10 years</td>
<td>1</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>11-15 years</td>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>16-20 years</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>21 years -more</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Teacher Certification</th>
<th>1st level</th>
<th>2nd level</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st level</td>
<td>3</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>2nd level</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Degree</th>
<th>Bachelor</th>
<th>Master</th>
<th>Doctor</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bachelor</td>
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<td>9</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Master</td>
<td>2</td>
<td>6</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Doctor</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>15</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

Although 317 South Korean elementary teachers participated in this survey, the participants had a right to skip questions. The total number of participants may differ among sections, which presents different demographic information.

Of the 317 survey respondents, 20 teachers volunteered to be interviewed for the study. The information about the interview participants is shown in Table 2. The detailed process of selecting participants and the procedures of the research is discussed in the following section.

4.3 Data Sources and Procedures

The results that are presented in this paper come from the following data sources: a web-based survey and a structured interview. Conducting a survey helped develop a broader perspective of elementary teachers’ knowledge for teaching mathematics. The survey method is useful when the purpose of the study is to quantitatively describe specific aspects of a given population [51]. If a survey obtains data based on a representative sample, the data can be generalizable to a population [52]. To insure validity and generalizability, this research randomly selected 1,109 South Korean elementary teachers, which resulted in 317 of them responding to the survey. The qualitative methods in this study consisted of interviews with 20 South Korean elementary teachers. This study focused on South Korean elementary teachers’ use of and beliefs about the subdomains of knowledge for teaching mathematics.

4.3.1 Survey

For the survey, a web-based questionnaire was developed, consisting of thirty-four items subdivided into four major sections: demographic, self-assessment, beliefs about mathematics, and application of mathematics knowledge for teaching. For the purpose of this study, only the survey questions that required respondents to rank order items are used for analysis. These survey items used the subdomains of knowledge for teaching mathematics from the conceptual framework. For example, survey respondents were asked, “What do you think is the most important mathematical knowledge elementary teachers should have?” They were then instructed to rank order a list of items (e.g., using manipulatives when teaching or using technology when teaching) according to most important mathematical knowledge.
for teachers. Since we assumed that the teachers might not be familiar with the academic terms for the subdomains of MKT (e.g., knowledge of mathematical representation), we provided an explanation or example for each subdomain (e.g., knowledge of mathematical representation—the knowledge related to presenting mathematics concepts to students using diverse verbal explanations, manipulatives, or illustrations, etc.). The instrument was translated and adapted into Korean by the authors. One author of this study, a native Korean speaker who is also fluent in English and has ten years of teaching experience in South Korea, participated in translating the survey items from English to Korean.

The researchers piloted the survey with a sample of ten members of the target population in May 2012. The reliability of the survey items is 0.8 based on test-retest assessment. In addition, ten specialists who work in South Korean elementary schools validated the terms that were used for the survey items. The teachers have master’s degrees in mathematics education as well as experience in developing elementary level textbooks based on the National Mathematics Curriculum. This process allowed the researchers to identify whether the respondents understood the instructions and questions and whether the meaning of the questions was the same for all respondents. Between June and August 2012, the link to the web-based questionnaire was sent to 1,109 elementary teachers by a school networking system in 30 randomly selected elementary schools in Seoul, South Korea. Participation was on a voluntary basis; there was no previous contact between researchers and respondents.

4.3.2 Interviews

There was a need for an in-depth investigation about what kind of knowledge for teaching mathematics South Korean elementary teachers use for mathematics instructions, as well as the reasons they use these knowledge subdomain, which may not be present in survey data. For this reason, interviews were conducted in order to understand teachers’ knowledge for teaching mathematics and reveal the connection between their knowledge and beliefs. For the structured-interview, questions were prepared to investigate the reasons for the orders of both teachers’ use of knowledge and beliefs about it. The participants were required to complete the survey questions, and follow-up interviews were conducted in order to understand the teachers’ intention.

4.4 Data Analysis

For this study, the main emphasis was to identify the subdomains of knowledge South Korean elementary teachers believe are important for effective mathematics instruction and the subdomains of knowledge they use when teaching mathematics. This section outlines the procedures applied to analysis of data generated from the survey results of 317 respondents and the reasons offered by 20 interviewed participants.

4.4.1 Statistical Analysis of Survey Results

To analyze the rank ordered data, the Randomized-Blocks ANOVA (RBANOVA) was employed. RBANOVA helps researchers decide whether the observed differences among group means are by chance or are systemic where the participants are the same in each group [53]. However, the researchers may not find where the difference lies with RBANOVA, although the RBANOVA provides overall differences among ranks. In order to have information that tells where differences exist, post hoc tests are conducted.

Based on the purpose of RBANOVA, there are two reasons for using the RBANOVA technique for this study. First, participants repeatedly ranked from group 1 (i.e. choice 1) to group 6. Second, there may be
dependencies among the ranks for each choice. That is, if the participant ranked choice 1 as first, other choices from choice 2 to choice 6 were not likely to be ranked as first. Additionally, the data satisfied the design requirements regarding the level of independent/dependent variables, assumption of normal distribution, and assumption of no outliers [53]; and the Huynh-Feldt correction was considered to satisfy sphericity assumptions.

Second, the RBANOVA indicates overall differences among ranks, which can evidence whether there is a statistical difference between any combinations of two ranks. In other words, a RBANOVA cannot specify where the difference lies. Tukey’s HSD tests with Bonferroni corrections were applied to examine the places where differences exist. In addition, the original rank ordered data was converted into scores to represent the importance of choices. The study also used the computer statistical tool SPSS (SPSS Statistics 20) to analyze the data; all results were reported at $\alpha=0.05$.

### 4.4.2 Descriptive Analysis of Interview Results

Based on the research question, we coded raw interview data by using the Excel program. The texts were coded on a line-by-line basis, since it is useful to represent data as objectively as possible in the early stages of data analysis [54]. Therefore, every line was assigned a short descriptor intended to represent its fundamental meaning. As noted previously, the interviews were conducted in order to provide an in-depth understanding on survey results rather than to analyze differences among teachers. Thus, results of the interview analysis were used in order to understand the findings from statistical analysis rather than to generate new codes or themes regarding South Korean teachers’ knowledge for teaching mathematics.

### 5. RESEARCH FINDINGS

This section summarizes the main findings emerging from the analysis of the data from the survey and interview sources. The presentation of the findings is organized around two primary areas: teachers’ use of knowledge for teaching mathematics and their beliefs about knowledge for teaching mathematics. For both areas, six subdomains of knowledge for teaching mathematics are identified: knowledge of mathematics content, mathematics curriculum, mathematics learners, mathematics education theory, mathematical representations, and mathematical technology.

#### 5.1 Survey Results

This section presents the results from analysis of survey data from the 317 respondents. To make sense of teachers’ rankings, the results of the RBANOVA with a Huynh-Feldt correction are presented for teachers’ use of knowledge for teaching mathematics and teachers’ beliefs about knowledge for teaching mathematics. Next, we present the South Korean elementary teachers’ hierarchy of their knowledge for teaching mathematics.

#### 5.1.1 Teachers’ Use of Knowledge

A RBANOVA, with a Huynh-Feldt correction, determined that teachers reported different priorities among the subdomains of knowledge for mathematics teaching, and the differences were statistically significant ($F(6.572, 1991.380) = 229.448, P < 0.001$), as shown in Table 3. Post hoc tests using the
Table 3. Results of Analysis of Variance of Teachers’ Use of Knowledge

<table>
<thead>
<tr>
<th>(I) Component</th>
<th>(J) Compared Component</th>
<th>Mean Difference I-J</th>
<th>Std. Error</th>
<th>Sig. b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge of mathematical technology (2.359*, .115**)</td>
<td>Knowledge of mathematics learners</td>
<td>-3.628</td>
<td>.160</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics representation</td>
<td>-668</td>
<td>.141</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics education theory</td>
<td>.684</td>
<td>.178</td>
<td>.002</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics content</td>
<td>-4.668</td>
<td>.150</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics curriculum (4.740*, .122**)</td>
<td>-2.382</td>
<td>.193</td>
<td>.000</td>
</tr>
<tr>
<td>Knowledge of mathematics learners (5.987*, .111**)</td>
<td>Knowledge of mathematics representation</td>
<td>2.961</td>
<td>.152</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics education theory</td>
<td>4.313</td>
<td>.169</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics content</td>
<td>-1.039</td>
<td>.148</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics curriculum</td>
<td>1.247</td>
<td>.182</td>
<td>.000</td>
</tr>
<tr>
<td>Knowledge of mathematics representation (3.026*, .101**)</td>
<td>Knowledge of mathematics education theory</td>
<td>1.352</td>
<td>.153</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics content</td>
<td>-4.000</td>
<td>.140</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics curriculum</td>
<td>-1.714</td>
<td>.183</td>
<td>.000</td>
</tr>
<tr>
<td>Knowledge of mathematics education theory (1.674*, .106**)</td>
<td>Knowledge of mathematics content</td>
<td>-5.352</td>
<td>.151</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics curriculum</td>
<td>-3.066</td>
<td>.166</td>
<td>.000</td>
</tr>
<tr>
<td>Knowledge of mathematics content (7.026*, .092**)</td>
<td>Knowledge of mathematics curriculum</td>
<td>2.286</td>
<td>.170</td>
<td>.000</td>
</tr>
</tbody>
</table>

Note: N=317, * Mean score for each subdomain. **Standard error.

Bonferroni correction revealed that all pair-wise comparisons were statistically significant at $\alpha = 0.05$.

5.1.2 Teachers’ Beliefs

An ANOVA with a Huynh-Feldt correction revealed that teachers’ beliefs about the subdomains of knowledge indicated different priorities, and those differences were statistically significant ($F(5.465, 1644.833) = 253.480, P < 0.001$) as shown in Table 4. Post hoc tests using the Bonferroni correction indicated that most pair-wise comparisons are statistically significant at an alpha level of .05, except the relationships between knowledge of mathematics learners and knowledge of mathematics curriculum as well as knowledge of mathematics representation and knowledge of mathematics education theory.

5.1.3 Hierarchy of Knowledge for Teaching

The next step in the analysis process was to establish teachers’ hierarchy of their knowledge for teaching. From the analysis of the survey responses, this study found that teachers have a certain order of importance they assigned to subdomains of knowledge for teaching mathematics regarding how they use mathematics knowledge in practice and what their beliefs about the subdomains of knowledge are. For easy interpretation and modeling of ranking of the subdomain of knowledge, we use a six-level inverted pyramid model to represent the hierarchy of knowledge for teaching mathematics. The underlying idea for using the inverted pyramid is that the literature has consistently shown that knowledge of content has represented the most common subdomains of knowledge for teaching across a variety of academic areas. Also, we selected the inverted pyramid to illustrate how the participants prioritized their use of and beliefs about the subdomains of knowledge.

A representative view of South Korean elementary teachers’ hierarchy of their use of the knowledge for teaching mathematics is shown in Figure 5. The teachers considered the subdomain Knowledge of mathematics content as the most important, when teaching mathematics. They also considered Knowledge
Table 4. Results of Analysis of Variance of Teachers’ Beliefs Toward Knowledge

<table>
<thead>
<tr>
<th>(I) Component</th>
<th>(J) Compared Component</th>
<th>Mean Difference (I-J)</th>
<th>Std. Error</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge of mathematical technology (1.321*, .088**)</td>
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<td>-3.467</td>
<td>.130</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics representation</td>
<td>-1.238</td>
<td>.114</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics education theory</td>
<td>-.821</td>
<td>.173</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics content</td>
<td>-4.685</td>
<td>.125</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics curriculum (4.711*, .098**)</td>
<td>-3.583</td>
<td>.141</td>
<td>.000</td>
</tr>
<tr>
<td>Knowledge of mathematics learners (4.788*, .100**)</td>
<td>Knowledge of mathematics representation</td>
<td>2.228</td>
<td>.144</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics education theory</td>
<td>2.646</td>
<td>.176</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics content</td>
<td>-1.21*</td>
<td>.132</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics curriculum</td>
<td>-.116</td>
<td>.153</td>
<td>1.000</td>
</tr>
<tr>
<td>Knowledge of mathematics representation (2.560*, .085**)</td>
<td>Knowledge of mathematics education theory</td>
<td>.417</td>
<td>.155</td>
<td>.113</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics content</td>
<td>-3.447</td>
<td>.133</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics curriculum</td>
<td>-2.344</td>
<td>.141</td>
<td>.000</td>
</tr>
<tr>
<td>Knowledge of mathematics education theory (2.142*, .115**)</td>
<td>Knowledge of mathematics content</td>
<td>-3.864</td>
<td>.147</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Knowledge of mathematics curriculum</td>
<td>-2.762</td>
<td>.154</td>
<td>.000</td>
</tr>
<tr>
<td>Knowledge of mathematics content (6.007*, .080**)</td>
<td>Knowledge of mathematics curriculum</td>
<td>1.103</td>
<td>.123</td>
<td>.000</td>
</tr>
</tbody>
</table>

Note: N=317, * Mean score for each subdomain. **Standard error.

Figure 5. Hierarchy of Teachers Use of the Subdomains Knowledge for Teaching Mathematics

The teachers’ use of knowledge for teaching mathematics is contrary to current research trends about teachers’ knowledge for teaching mathematics. As noted above, most of these studies have focused on pedagogical transformation of mathematics concepts [55]. However, the teachers considered Knowledge of mathematics content and Knowledge of mathematics learners to have priority over Knowledge of representations, the subdomain in the fourth position. It seems logical that the teachers ranked Knowledge of mathematical representations after Knowledge of mathematics content or Knowledge of mathematics learners, considering that mathematical representation indicates the way of transforming mathematics
concepts to students’ level [13].

Next, the ranking of the teachers’ beliefs about the subdomains for knowledge for teaching mathematics were examined. It is noteworthy that the teachers’ beliefs about the subdomains of knowledge for teaching mathematics were ranked slightly different in comparison to their actual use of the subdomains as shown in Figure 6. The dashed line between Knowledge of mathematics curriculum and Knowledge of mathematics learners, as well as Knowledge of mathematical representations and Knowledge of mathematics education theory indicates the lack of statistical significance between the means for each subcategory (p = .05).

Although the overall ranking of the subdomains of knowledge are similar to the previous model, there are differences between them. The order of Knowledge of mathematics learners and Knowledge of mathematics curriculum as well as Knowledge of mathematics technology and Knowledge of mathematics education theory changed as shown in Figure 7.

We are not sure what caused this difference between the actual use of knowledge and the beliefs about knowledge for teaching mathematics in this study. One of the possible explanations is that the teachers place theoretical knowledge, which provides a basis for their instruction, such as curriculum or theory of mathematics education as a priority above practical knowledge, such as the use of technology.

5.2 Interview Results

From the 317 respondents to the survey questions, we obtained useful information about their priority of the importance of the knowledge for teaching mathematics regarding use in practice and beliefs. However, to provide a way to understand the meaning behind the information presented by the respondents about the subdomains of knowledge for teaching mathematics, we interviewed 20 elementary teachers. The primary questions were to inquire of their reasons for ranking the subdomains of knowledge as they did. The major findings from analysis of the interview data are presented in this section.
5.2.1 Teachers’ Use of Knowledge for Teaching Mathematics

Eighteen of twenty teachers provided similar rankings of the subdomains of knowledge (Table 5). However, the two remaining teachers only changed the order between Knowledge of mathematics curriculum and Knowledge of mathematical representations. As noted previously, the interviews were conducted in order to provide an in-depth understanding of survey results rather than to analyze differences among teachers. Thus, this study only presents the results of the interviews of 18 participants, who ranked the subdomains of knowledge in the same ways as the survey respondents. The results of interviews are presented in a descriptive way. Table 5 includes the participants’ rationales for their ranking of the subdomains of knowledge for teaching mathematics.

The majority of the teachers ranked Knowledge of mathematics content and Knowledge of mathematics learners higher than the other subdomains of knowledge, stating that to teach mathematics, they needed to know the content. Also, they reasoned that knowing content and concepts was the best way to support their students. It is not surprising that the majority of the teachers interviewed gave their lowest ranking to Knowledge of mathematical technology and mathematics educational theory because they stated that they did not use technology when teaching or that they could teach mathematics without relying on technology. They offered similar explanations about the use of educational theory when teaching mathematics. When teaching mathematics, the teachers suggested that they could teach mathematics without knowledge of educational theory.

5.2.2 Teachers’ Beliefs about Knowledge for Teaching Mathematics

Sixteen of twenty teachers ranked the items in the same way as illustrated in Table 6. However, the four remaining teachers ranked two subdomains of knowledge differently. According to the purpose of the interview, this section only presents the analysis results of interviews of 16 participants, who ranked the subdomains of knowledge to survey respondents. Table 6 illustrates the participants’ rationales for the ranking regarding their beliefs about the subdomains of knowledge for teaching mathematics.

When discussing their beliefs about the subdomains of knowledge for teaching mathematics, the teachers interviewed offered similar explanations to the ones presented regarding their use of the subdomains of knowledge. However, in several cases, their explanations included more details. For example, knowledge
Table 5. Results of Interviews Regarding Teachers’ Use of Knowledge

<table>
<thead>
<tr>
<th>Rank Order</th>
<th>Subdomains of Knowledge</th>
<th>Rationales ()*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Knowledge of mathematics content</td>
<td>I cannot teach mathematics without knowing what I am going to teach. (15) Without knowing the concepts, I cannot support my students to learn about it. (12)</td>
</tr>
<tr>
<td>2</td>
<td>Knowledge of mathematics learners</td>
<td>I should know my students’ level in order to explain the mathematics concept (16). I design my lesson according to my students’ characteristics such as mathematics background or preferences for activities (13).</td>
</tr>
<tr>
<td>3</td>
<td>Knowledge of mathematics curriculum</td>
<td>I can find students’ learning history on mathematics based on the sequence of the National Mathematics Curriculum (14). I need to know students’ sequence of learning in order to prevent using mathematics terms, which my students have not yet learned (10).</td>
</tr>
<tr>
<td>4</td>
<td>Knowledge of mathematical representations</td>
<td>If I know the mathematics concepts and characteristics students, I may develop my own ways of representing mathematics concepts. Thus, it is not as significant as the other three subdomains of knowledge, which ranked one to three (14). I think that I do not use this knowledge often (5).</td>
</tr>
<tr>
<td>5</td>
<td>Knowledge of mathematical technology</td>
<td>I do not use knowledge of mathematics technology often in my mathematics classroom (17). I can teach mathematics without technology. (14).</td>
</tr>
<tr>
<td>6</td>
<td>Knowledge of mathematics educational theory</td>
<td>I do not know mathematics educational theory well, but I do not see any problems when I teach mathematics (17). The other subdomains are more significant than this subdomain (3).</td>
</tr>
</tbody>
</table>

*Note. The number in () represents the numbers of participants providing similar reasons.

of educational theory and mathematics technology again received lower rankings but switched positions. Although they did not use technology during their instruction, when following the objectives of the lesson, they believed that teachers should know this subdomain. However, they had a difficult time articulating why they believed it was important. They were able to offer explanations about the value of educational theory, which is ranked last. For example, as illustrated in Table 6 about half the teachers interviewed stated, “Only designers of curriculum, textbooks, or teachers’ guidebooks need mathematics educational theory.

We also wanted to compare the interview participants’ rankings to those of the 317 survey respondents. From the results of statistical analysis of the survey data, we found that South Korean teachers ranked their use of knowledge for teaching mathematics differently from their beliefs about knowledge for teaching mathematics. From the analysis of the interview data, we found that fifteen of twenty teachers who were interviewed ranked the subdomains of knowledge differently as well. Table 7 illustrates the participants’ rationales for their ranking of the subdomains of knowledge.

When taking a closer examination of the teachers’ rationales regarding their beliefs in which they compared one subdomain to another, the teachers interviewed were able to articulate why they believed that knowledge of educational theory was more important than use of educational technology. Sometimes, their explanations seemed to present a contradiction. For example, one teacher offered the following elaborated explanation:

Although I use knowledge of mathematical technology more than my knowledge of mathematics educational theory in my mathematics instruction, I believe that teachers should know theory more than technology. Although I acquired knowledge of mathematical technology from teacher education programs, I need to know how to apply it based on my students’ characteristics. Thus, to have knowledge of mathematical technology does not ensure that I may use the knowledge in my mathematics instruction.
Table 6. Results of Interviews Regarding Teachers’ Beliefs about Knowledge

<table>
<thead>
<tr>
<th>Rank Order</th>
<th>Subdomains of Knowledge</th>
<th>Rationales ()*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Knowledge of mathematics content</td>
<td>Teachers cannot teach mathematics without knowing what they are going to teach. (15). Teachers should know the concepts before they teach in order to support their students to learn about it (9). If teachers do not know the mathematics concepts, they cannot teach it. Thus, teachers should consider whether or not they know the concept well (8).</td>
</tr>
<tr>
<td>2</td>
<td>Knowledge of mathematics curriculum</td>
<td>Teachers should know what students already learned. Knowing sequence of the mathematics topic based on the mathematics curriculum may provide understanding of students’ learning history (14). To prevent using mathematics concepts, which students learn in another grade, teachers should know the sequences of students’ learning based on the mathematics curriculum (11). Knowledge of mathematics curriculum may provide certain criteria to understand students’ mathematical background (8).</td>
</tr>
<tr>
<td>3</td>
<td>Knowledge of mathematics learners</td>
<td>Teachers should know their students in order to explain the mathematics concept according to students’ level (15). The lesson plan might be changed according to the students’ characteristics, such as mathematics background or preferences for activities (13). Without understanding students’ characteristics as mathematics learners, teachers may not teach mathematics effectively (6).</td>
</tr>
<tr>
<td>4</td>
<td>Knowledge of mathematical representations</td>
<td>If teachers know the mathematics concepts and characteristics of students, they may develop their own ways of representing mathematics concepts. Thus, it is not as significant as the other three subdomains of knowledge (14). It would be meaningless if teachers have knowledge of mathematical representation, but do not know how to use it based on students’ level and mathematics concepts (8).</td>
</tr>
<tr>
<td>5</td>
<td>Knowledge of mathematical technology</td>
<td>I do not believe that teachers must have this knowledge. It would be better for teacher if they have this knowledge. However, it would not matter if teachers do not have this knowledge (15). Teachers may teach mathematics without technology. (14) Teachers use technology selectively according to objectives of the lesson. However, there are diverse alternatives ways of presenting mathematics concepts. (10)</td>
</tr>
<tr>
<td>6</td>
<td>Knowledge of mathematics educational theory</td>
<td>It would be helpful for teachers if they know the mathematics educational theory. However, teachers can teach mathematics well without theory (16). Teachers do not need to know mathematics educational theory. Only designers of curriculum, textbooks, or teachers’ guidebooks need mathematics educational theory (9).</td>
</tr>
</tbody>
</table>

*Note. The number in () represents the numbers of participants providing similar reasons.

About half of the teachers interviewed offered a similar construction in which they expressed a strong belief about a subdomain of knowledge but did not apply it during their instructional practices.

6. DISCUSSION OF FINDINGS

This section summarizes the main findings emerging from the analysis of the data from the survey and interview sources. The presentation of the findings is organized around two primary areas: teachers’ use of knowledge for teaching mathematics and their beliefs about knowledge for teaching mathematics. For both areas, six subdomains of knowledge for teaching mathematics are identified: knowledge of mathematics content, mathematics curriculum, mathematics learners, mathematics education theory,
Table 7. Results of Interviews Regarding Teachers’ Beliefs about Knowledge

<table>
<thead>
<tr>
<th>Changes of Order</th>
<th>Rationales ( )*</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Use of Knowledge] Knowledge of mathematics curriculum and Knowledge of mathematics learners</td>
<td>Although I use knowledge of mathematics learners more than knowledge of mathematics curriculum in my instruction (since I need to interpret mathematics concepts according to students’ level), I may acquire my knowledge of students while I am teaching. However, I should know the curriculum before I teach mathematics (13). I may acquire curriculum knowledge from teacher education programs. However, I may not acquire knowledge of my students from them. Thus, I believe that teachers know about curriculum before they teach, but knowledge of mathematics learners might be formed from teaching experiences (11).</td>
</tr>
<tr>
<td>[Beliefs about Knowledge] Knowledge of mathematical technology and Knowledge of mathematics educational theory</td>
<td>Although I use knowledge of mathematical technology more than my knowledge of mathematics educational theory in my mathematics instruction, I believe that teachers should know theory more than technology. Although I acquired knowledge of mathematical technology from teacher education programs, I need to know how to apply it based on my students’ characteristics. Thus, to have knowledge of mathematical technology may not ensure that I may use the knowledge in my mathematics instruction. (10). It is worth more to have knowledge of mathematics educational theory than mathematical technology, since technology is more about skill than the basis for mathematics instruction (3).</td>
</tr>
</tbody>
</table>

*Note. The number in () represents the numbers of participants providing similar reasons.

mathematical representations, and mathematical technology.

6.1 Survey Results

Findings suggest that South Korean elementary teachers’ beliefs about and use of the subdomains of knowledge for teaching are not limited to one specific area. The South Korean elementary teachers’ hierarchy of knowledge for teaching mathematics provides some clues about the kind of curriculum that might be provided for teachers to develop their knowledge from pre and in-service teacher education programs. Unlike the past studies that focus on teachers’ knowledge regarding transformation of mathematics concepts, this present study found that teachers use their knowledge of mathematics concepts as well as their beliefs about mathematics content in setting their priority for instruction.

So far, the significance of teachers’ knowledge of mathematics content has been overlooked compared to the knowledge of pedagogical transformation of mathematics concepts. However, as Ma [56] points out, understanding mathematics content, which includes mathematics concepts, is not easy for teachers because the mathematics concepts characterize both the mental representations that are originated from physical experiences and the mental network among the representations [57]. For example, students may develop the mental representation of a triangle based on their experiences. The mental representation of a triangle becomes more sophisticated by connecting with the other mathematics concepts such as a concept of angle, square and circle. By comparing similarities and differences with the other shapes, students may further develop their conceptual understanding of a triangle as shown in Figure 8.

Students who understand these mathematics concepts may generate new mathematical knowledge rather than relying on memorization of terms only [58]. Thus, teachers should know mathematics concepts thoroughly in order to support their students to develop mathematics concepts from physical experiences and to develop the mental network among the representations in the students’ own mind [57].

South Korean teachers chose Knowledge of mathematics learners as the second most important subdomain of knowledge that teachers need to use for instructional purposes. Students, as mathematics learners, have diverse characteristics in terms of intelligence as well as their attitude. Therefore, there needs to be more investigations about what teachers should know about mathematics concepts and
characteristics of students for an effective mathematics instruction. In addition, the recent research regarding Knowledge of mathematical representation should be re-examined in terms of mathematical concepts and characteristics of students.

It is interesting that the teachers consider that Knowledge of mathematics curriculum is much more important than Knowledge of mathematical representations. Studies point out that teachers should know how to take advantage of curriculum for supporting students’ development of mathematical understanding [59]. The importance of curriculum knowledge is overlooked [60], and the definition of curriculum knowledge is still ambiguous [23]. This study assumes that Knowledge of mathematics curriculum is the sequence of the mathematics topics in the national curriculum both between and within grades for teaching. If the teachers consider the Knowledge of mathematics curriculum as one of the vital domains of their knowledge, it is suggested that further studies be conducted with diverse teachers to understand Knowledge of mathematics curriculum.

The teachers ordered the knowledge of representation in the fourth position. However, it may be reasonable for the teachers to put the knowledge of representation after knowledge of mathematics concepts or mathematics learners, when considering that mathematical representation characterizes the way of transforming mathematics concepts to students’ level. However, recent studies focus on the specific scenes of mathematics instruction, including mathematical representation, in order to define teachers’ knowledge for teaching mathematics, overlooking the relationship with mathematics concepts at the students’ level (e.g. [35, 61, 62]). The lack of connections among the subdomains of knowledge for teaching mathematics seems to place limitations on the effect of elementary teachers’ mathematical knowledge. While research suggested that elementary teachers’ mathematical knowledge did have some effect on student mathematics achievement, elementary teachers’ mathematical knowledge had little effect on students who received low marks from previous achievement tests (e.g. [7, 14, 63]). Thus, it is suggested that teachers’ Knowledge of mathematics representation should be understood in the relationship among the other subdomains of knowledge for teaching mathematics.

Although it is revealed that South Korean elementary teachers consider Knowledge of mathematics education theory and Knowledge of mathematical technology as the least important, it does not indicate that these subdomains of knowledge are less important than the other subdomains of knowledge. The South Korean elementary teachers’ hierarchy of the knowledge for teaching mathematics only presents the order of subdomains of knowledge, which South Korean teachers use in their mathematics instruction. This does not indicate the significance of each domain in mathematics instruction. We do not know if
the teachers use all domains of knowledge in their teaching practices or choose selectively. Also, there needs to be more investigations focusing on the other subdomains of knowledge such as mathematics content and mathematics learners, which the Korean elementary teachers consider most important during instruction.

7. CONCLUSIONS

As recent research studies have revealed, teachers’ mathematical knowledge can influence student learning (e.g. [39]), increasing the interest in teachers’ Mathematics Knowledge for Teaching. However, there still remains underspecified aspects of teachers’ knowledge for teaching mathematics, although there have been various attempts to reveal teachers’ knowledge for teaching mathematics as presented above [11]. One of the major reasons is that current studies may focus on a couple of subdomains of elementary teachers’ MKT by applying a framework, which was developed by a few conceptual studies (e.g. [12, 23]). Specifically, according to Da Ponde and Chapman [54], for the last two decades most of these studies only focused on pedagogical transformation of mathematics concepts, which is only one of the subdomains of teachers’ knowledge.

Since recent studies only focus on the specific aspects of mathematics instruction and emphasize how teachers represent mathematics concepts, this research overlooks the relationship between pedagogical content knowledge and mathematics concepts or students’ level of understanding (e.g. [36, 61, 62]). The lack of connections among the subdomains of knowledge for teaching mathematics exposed limitations regarding the affect of elementary teachers’ mathematical knowledge. While elementary teachers' mathematical knowledge did have some affect on student mathematics achievement, it had little affect on students who received low marks from previous achievement tests (e.g. [7, 14, 63]). It is suggested that teachers’ knowledge of mathematics representations should be understood in relationship to the other subdomains of knowledge for teaching mathematics. Also, there needs to be more investigation focusing on the other subdomains of knowledge, such as mathematics content and knowledge of mathematics learners, which the Korean elementary teachers consider most important during instruction.

The focus of the studies on one subdomain of teachers’ knowledge for teaching mathematics may inhibit us from having a better understanding of teachers’ knowledge for teaching mathematics. Additionally, there is a lack of detailed information about the other subdomains of teachers’ knowledge for teaching mathematics (e.g. knowledge of mathematics curriculum). In order to broaden the perspectives of elementary teachers’ knowledge for teaching mathematics, this study focuses on elementary teachers of South Korea. International research may be useful to illustrate diverse mechanisms by which teaching and learning are related and the processes by which students construct meaning from classroom instruction [64, 65]. Results from international studies might provide some clues about elementary teachers’ knowledge for teaching mathematics that would otherwise have been missed. However, the ways that South Korean elementary teachers apply their knowledge to mathematics instruction, which is based on mathematics curriculum standards and content used in the United States, may offer meaningful information about elementary teachers’ MKT in the United States.

The findings of this study provide implications of how policymakers might develop pre-service and in-service elementary mathematics teacher education programs. In many countries, careful consideration of effective pre-service mathematics teacher preparation is evident (e.g. [65]). Therefore, it is important to find effective ways to educate mathematics pre-service teachers to improve the capabilities of mathematics teacher preparation programs as well as for long-term perspectives of the development of the national mathematics education system. High-quality mathematics elementary school teachers are needed because the fundamental concepts of mathematics are introduced to students during their elementary school grades.
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